Harnack Inequality for Second Order Linear Ordinary Differential Inequalities

An Honors Thesis (HONR 499)

by

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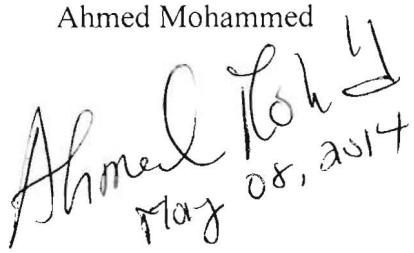

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Abstract

The Harnack Inequality is an important tool in the study of qualitative properties of solutions to elliptic and degenerate elliptic partial differential equations. Perhaps the first paper that discusses such an inequality for ordinary linear differential equations of second order is the article *A Harnack Inequality for Ordinary Differential Equations* [1]. The objective of this project was to extend the result of [1] to the ordinary differential inequality $L[u] \leq 0$, where $L[u] = u'' + p(x)u' + q(x)u$. We continued to extend this results by studying a Harnack-type inequality for $L[u] \leq f(x)$. To achieve this, we used some well-established facts on Maximum Principles from Protter and Weinberger's book [2]. We provide complete proofs for some of these results that are different from those presented in [2].

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